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Disentanglement of Source and Target and the Laser Quantum State

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Disentanglement of a laser source from its target qubit is proposed as a criterion establishing the laser quantum state as a coherent state. It is shown that the source-target density operator has a *unique* factorization in coherent states when the environmental record monitoring laser pump quanta is ignored. The source-target state conditioned upon the *complete* environmental record is entangled, though, as a state of known total quanta number (source plus target).

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We write this letter in response to two recent debates, which, although they arose and were carried on independently, are shown here to be intimately connected. The first traces its origin to the observation of Javanainen and Yoo [1] (see also [2, 3]) that the detection of atoms in overlapping Bose-Einstein condensates produces an interference pattern "even though the condensates are taken to be in number states with no phases whatsoever;" thus, it is not necessary for each condensate to be assigned a wavefunction of definite phase—specifically, for example, a coherent state. Subsequently, Mølmer [4] argued for a rather provocative extrapolation, directed this time at optical rather than material coherence: "We conjecture that optical coherences, ..., do not exist ...." and "A conclusion of this paper is that it does not matter whether coherence exists or not; observable phenomena in optics and quantum optics are unchanged, and in this way optical coherences may be regarded as a convenient fiction." Elaborated a little, Mølmer asserts that conventional sources of optical coherence—lasers—do not (cannot in fact) produce coherent states, though assigning one is harmless enough—it yields correct answers—and, in particular, such assignment is "convenient" since the more valid (nonfictional) calculation requires one to work with entangled states.

Unsurprisingly, not all readers of Mølmer agreed [5, 6]. In retrospect, however, his and Javanainen and Yoo's comments are not as new or radical as it seems. If, to simplify, we focus the issue on whether number states or coherent states give the more fundamental account of laser light, then the question is raised already by the earliest laser theories: the Scully-Lamb master equation [7] describes a birth-death evolution between number states—$\ldots|n-1\rangle \rightarrow |n\rangle \rightarrow |n+1\rangle \rightarrow |n\rangle \ldots$ as a stochastic process—while the phase-space formulations of the Lax and Haken schools [8, 9] underlie the popular picture of a stochastically evolving coherent state, $|\alpha_t\rangle$, with α_t a Brownian path mapped out in the complex plane.

In a final chapter, this first debate was carried into the quantum information field by Rudolph and Sanders [10], who criticize the claimed implementation of continuous variable quantum teleportation (CVQT) by Furusawa *et al.* [11]. They argue that "genuine CVQT cannot be achieved using conventional laser sources, due to an absence of intrinsic coherence" and for this cite the assertion of Mølmer [4] (see [12] for a response).

The second, related debate arose within the quantum information field itself, initiated by Geo-Banacloche [13] and van Enk and Kimble [14] who point out that the state of a laser manipulating a qubit is (or potentially is) entangled, in which case decoherence arising from the resolution of the entanglement (tracing over the laser) must be considered when assessing the fidelity of the manipulation. Mølmer, in the quoted paper, makes a similar point: "the state of an atomic target for a laser beam is entangled with the field states, as inferred from the Hamiltonian (1)." From the other side, Itano [15] objects, arguing, principally, following Mollow [16], that the Hamiltonian of a two-state system interacting with a quantum field in a coherent state—say $|\alpha_t\rangle$—may be taken with operators replaced by classical parameters, e.g., the annihilation operator by α_t. He asserts, in particular, that no source of decoherence is missed when this is done; *all* decoherence of the qubit arises through spontaneous emission (see [17] and [18] for replies to Itano's comment).

This letter is most directly an extension of the contribution of one of us to the second debate. Taking a different approach to Itano, Nha and Carmichael [19] use the quantum trajectory theory of cascaded open systems [20] to reach his conclusion; assuming the laser produces a coherent state in the absence of the target qubit—say $|\alpha_t\rangle$—they show that a factorized state is reached in its presence, a state $|\alpha_t\rangle \otimes |T(t)\rangle$, with $|T(t)\rangle$ the target state. Here, building upon this result, we bring the two debates together. First, Nha and Carmichael's *assumption* of a coherent state is relaxed; the laser is modeled realistically in the spirit of [7, 8, 9], though the work of Rice and Carmichael [21] outlines, more simply, the essential features of the laser model. We begin then with the question: does a *realistic* laser source entangle with its target qubit? We argue, yes, but only so long as the source-target state is conditioned upon the entire environmental record, including the record of the laser pumping (we adopt the notion of entangled conditional states [22]). Discarding the pumping record, so the source-target state is mixed, the conclusion does not follow. Rather we show that the resulting mixed state is factorizable, and, if the source is classical (possesses a nonsingular and positive Glauber-Sudarshan P func-

tion), the factorization is *unique* and assigns the state of the laser source as a coherent state.

Before we elaborate these results a comment is in order regarding the nature of the problem we aim to address and our strategy for an answer. Griffiths [23] states the problem as well as it can be stated: "In essence, non-relativistic quantum mechanics consists in solving the Schrödinger equation and giving a physical interpretation to the solutions (including boundary and initial conditions). The former is a mathematical problem about which there is little disagreement. The latter has given rise to an extended controversy which is far from being resolved." We might add that much of the controversy stems from a mixing of the former with the latter, and emphasize that interpretation must, indeed, be *given*; the Schrödinger equation will not serve up the state of laser light on its own: even on a cosmic scale it merely entangles—and nothing ever happens (it says nothing of events). We quote from Griffiths' paper on consistent histories (see also Griffiths [24], Omnès [25], and Gell-Mann and Hartle [26]). Without directly building upon that program, we understand the problem of interpretation much as it does, see histories—trajectories (sequences of events or assignable properties)—as the key element of an answer, and, most importantly for this letter, place *consistency* at the center of our argument.

The apparent consensus in quantum optics is that the question of the state of laser light "ends up being of only academic interest" [6] and "which way one chooses to resolve a particular density operator—i.e., which states one chooses to ascribe to individual realizations of the ensemble—may ultimately be little more than a matter of convenience" [5]; to claim otherwise is a commission of the "partition ensemble fallacy" [10]. Our claim in this letter *is* otherwise. We place the question in the category of those addressed by decoherent histories, not merely academic, but of deep importance to a coherent formulation of quantum mechanics. We claim, moreover, that criteria exist to assign the laser state uniquely as a coherent state. The main points of our argument, elaborated below, are: (i) the output of a light source is intended for illuminating a target, therefore the principal criterion in assigning a state is to disentangle the source and target so the behavior of the latter may be seen to follow from properties thus assigned the former; (ii) such disentanglement is not always feasible but is for any classical photoemissive source if the description is suitably coarse-grained—i.e., if information in the environment allowing for the tracking of source-target correlations at the one-quantum level is discarded; (iii) coherent states provide the only self-consistent disentanglement; by this we mean that with the factorization adopted, a detailed description of the target response can be given, as a stochastic process, preserving the factorization at every step.

While (i) is central, (iii) is also essential to escape the partition ensemble fallacy, which, from [10], states: "Within the framework of standard quantum mechanics, the density operator is the complete description of the quantum state [10], and there is no reason to accord preferential treatment to one particular decomposition of the infinite number of equivalent decompositions for a mixed state." Against this "behavior" in (i) and "stochastic process" in (iii) point to the fundamental oversight of the quoted "fallacy": the density operator *does not* provide a complete description of laser light as a quantum field; an infinity of correlation functions do that. We assert that coherent states disentangle the laser source and target not only in their state $\rho(t)$, but as a stochastic process, correlation functions to all orders taken into account.

We build our argument around the model scattering scenario depicted in Fig. 1 which we describe by the master equation in Lindblad form [20, 27, 28]

$$\frac{d\rho}{dt} = (\mathcal{L}_S + \mathcal{L}_T + \mathcal{L}_{ST})\rho, \tag{1}$$

where $\mathcal{L}_S$ depends upon the particular laser model used and (setting $\hbar = 1$)

$$\mathcal{L}_T = -i[\hat{H}_T, \cdot\,] + \tfrac{1}{2}(2\hat{s}\cdot\hat{s}^\dagger - \hat{s}^\dagger\hat{s}\cdot - \cdot\,\hat{s}^\dagger\hat{s}), \tag{2}$$

$$\mathcal{L}_{ST} = -i[\hat{H}_{ST}, \cdot\,] + \tfrac{1}{2}(2\hat{f}\cdot\hat{f}^\dagger - \hat{f}^\dagger\hat{f}\cdot - \cdot\,\hat{f}^\dagger\hat{f}), \tag{3}$$

with coupling Hamiltonian

$$\hat{H}_{ST} = i\tfrac{1}{2}\sqrt{\gamma_S\gamma_T^f}(\hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger), \tag{4}$$

and jump operators

$$\hat{f} = \sqrt{\gamma_S}\hat{a} + \sqrt{\gamma_T^f}\hat{b}, \qquad \hat{s} = \sqrt{\gamma_T^s}\hat{b}; \tag{5}$$

operator $\hat{a}$ annihilates photons from the laser mode and $\hat{b}$ lowers the target qubit. The strength of the one-way coupling is specified through bandwidths, γ_S and γ_T^f, of the source output channel and target input channel, and the target scatters light with Einstein A coefficient $\gamma_T = \gamma_T^f + \gamma_T^s$ (forwards plus side); see van Enk [29] for the generalization of this simplified two-channel model.

The one-way nature of the coupling (scattering character) is made explicit in a quantum trajectory unraveling of Eq. (1) [20], which replaces $\hat{H}_{ST}$ by the non-Hermitian Hamiltonian

$$\hat{H}_{ST} - i\tfrac{1}{2}\hat{s}^\dagger\hat{s} - i\tfrac{1}{2}\hat{f}^\dagger\hat{f} = -i\sqrt{\gamma_S\gamma_T^f}\hat{a}\hat{b}^\dagger - i\tfrac{1}{2}\gamma_S\hat{a}^\dagger\hat{a} - i\tfrac{1}{2}\gamma_T\hat{b}^\dagger\hat{b}. \tag{6}$$

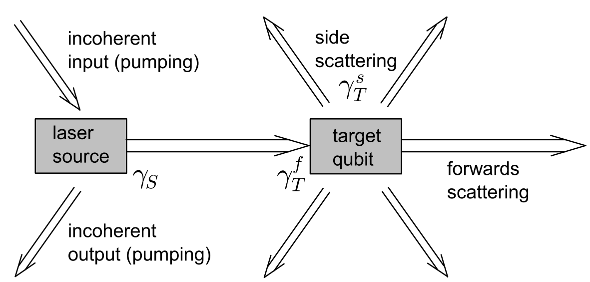

FIG. 1: Schematic of the scattering scenario for a laser source illuminating a target qubit.

It is essential as it then follows from Eqs. (5) and (6), assuming a coherent state, $|\alpha_t\rangle$, for the source, that the conditional state *factorizes* as [19]

$$|\psi_{\rm REC}(t)\rangle = |\alpha_t\rangle \otimes |T(t)\rangle, \tag{7}$$

with $|T(t)\rangle$ the target state. The result requires the cancelation of $\hat{a}^\dagger\hat{b}$ from $\hat{H}_{ST}$ and the special status of coherent states as eigenstates of $\hat{a}$.

Now let us remove the coherent state assumption and allow the laser to be described, dynamically, through $\mathcal{L}_S$. Generally, details of what follows depend on the laser model. We specify it here in the simplest form to accommodate our aims—i.e., as a birth-death process for carrier (gain medium) and photon (laser light) numbers N_t and n_t [21]. A central feature is the *incoherent* pumping. At this input-output channel (Fig. 1) the passage of energy quanta into and out of the laser resonator may be tracked and corresponding changes in N_t inferred. As a simplifying feature, we assume the gain-medium polarization may be adiabatically eliminated so n_t changes according to a birth-death process too: e.g., under stimulated emission, $(N_t, n_t) \to (N_t - 1, n_t + 1)$. If, then, the side and forwards scattering of the target qubit is monitored, thus completing the scattering record, the conditional state is an *entangled* state of photon number and qubit excitation,

$$|N_t\rangle \otimes \left[a(t)|n_t\rangle \otimes |-\rangle + b(t)|n_t - 1\rangle \otimes |+\rangle\right], \tag{8}$$

where $a(t)$ and $b(t)$ are real coefficients.

At this point our rudimentary model might be criticized as, for example, it rules out entanglement of the laser medium and its emitted light. Recall, however, that breaking the unending chain of entanglement is the goal, and something recognized as necessary—if named a "convenient fiction"—also by Mølmer [4]. Recall that interpretation must be *given*. We argue in this spirit that coherent states are special, indeed unique: they assign properties, separately and self-consistently, to the laser source and target qubit.

We now discard the pumping record that permits the labeling of conditional state (8) by definite numbers, N_t and n_t; although, in principle, the tracking of every quantum in the environment might be permitted, it is certainly impossible to achieve in practice. Thus, we now consider a coarse-grained description, summing over possible carrier numbers to arrive at a source-target density operator of the form

$$\rho(t) = \sum_{N=0}^{\infty} p_N(t)\rho^{(N)}(t), \tag{9}$$

where $\rho^{(N)}$ has non-zero matrix elements [Eq. (8)]

$$\rho^{(N)}_{n,-;n,-}, \quad \rho^{(N)}_{n-1,+;n-1,+}, \quad \rho^{(N)}_{n,-;n-1,+}, \quad \rho^{(N)}_{n-1,+;n,-}. \tag{10}$$

The question follows: do these equations specify an entangled or a factorizable state? The answer by construction is a factorizable state, since the same set of nonzero elements is recovered by the expansion

$$\rho^{(N)} = \frac{1}{2\pi}\int P(r)|re^{i\phi}\rangle\langle re^{i\phi}| \otimes |T_\phi\rangle\langle T_\phi| r dr d\phi, \tag{11}$$

where $|re^{i\phi}\rangle$ is a coherent state of amplitude r and phase ϕ, $P(r)$ is a distribution over r, and

$$|T_\phi\rangle = a' e^{-i\phi/2}|-\rangle + b' e^{i\phi/2}|+\rangle \tag{12}$$

is a target state. The phase average is the crucial thing, and it suggests that the principal function of the entangled state (8) is to account for a phase correlation between source and target when the known numbers N_t and n_t disallow a separable phase assignment. If, on the other hand, the pumping record is discarded, the correlation between n_t and qubit excitation is discarded too. Only the phase correlation remains—the principal signature of the macroscopic physics. This, coherent states can capture in an assigned property of the source—complex amplitude $re^{i\phi}$—and a correlated target response. Such an interpretation realizing the separation of source and target is clearly desired, if not required, since (9) is indeed a factorizable state. It remains to show its generality and its uniqueness.

With Eq. (11) as our motivation, we aim to show that the solution, $\rho(t)$, to master equation (1) factorizes in coherent states whenever the source is classical—i.e., whenever its density operator possesses a nonsingular and positive Glauber-Sudarshan P representation. To this end, we regroup terms in the superoperator $\mathcal{L}_S + \mathcal{L}_T + \mathcal{L}_{ST}$ reexpressing it as $\mathcal{L}'_S + \mathcal{L}'_T + \mathcal{L}'_{ST}$, where $\mathcal{L}'_S$ and $\mathcal{L}'_T$ operate on the source or target only,

$$\begin{aligned}\mathcal{L}'_S &= \mathcal{L}_S + \tfrac{1}{2}\gamma_S(2\hat{a}\cdot\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\cdot - \cdot\,\hat{a}^\dagger\hat{a}), &(13)\\ \mathcal{L}'_T &= -i[\hat{H}_T, \cdot\,] + \tfrac{1}{2}\gamma_T(2\hat{b}\cdot\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\cdot - \cdot\,\hat{b}^\dagger\hat{b}), &(14)\end{aligned}$$

and all cross terms are collected in the coupling

$$\begin{aligned}\mathcal{L}'_{ST} &= -i[\hat{H}_{ST}, \cdot\,] + \tfrac{1}{2}\sqrt{\gamma_S\gamma_T^f}(2\hat{a}\cdot\hat{b}^\dagger + 2\hat{b}\cdot\hat{a}^\dagger \\ &\quad -\hat{a}^\dagger\hat{b}\cdot - \hat{b}^\dagger\hat{a}\cdot - \cdot\,\hat{a}^\dagger\hat{b} - \cdot\,\hat{b}^\dagger\hat{a}) \\ &= \sqrt{\gamma_S\gamma_T^f}(\hat{a}\cdot\hat{b}^\dagger + \hat{b}\cdot\hat{a}^\dagger - \hat{b}^\dagger\hat{a}\cdot - \cdot\,\hat{a}^\dagger\hat{b}). \qquad (15)\end{aligned}$$

We also need the master equation traced over the qubit,

$$\frac{d\rho_S}{dt} = \mathcal{L}'_S\rho_S, \qquad \rho_S = \mathrm{tr}_T(\rho), \tag{16}$$

assumed to possess the solution for a classical source

$$\rho_S(t) = \int P(\alpha, \alpha^*, t)|\alpha\rangle\langle\alpha| d^2\alpha, \tag{17}$$

where $P(\alpha, \alpha^*, t)$ is a nonsingular and positive function; most likely the solution is coarse-grained—e.g., through a system size expansion—but it is acceptable in the spirit of the discarded pumping record. For simplicity, all dependence on the gain medium—e.g., $|N_t\rangle$ in (8)—is omitted.

Note now that $P(\alpha,\alpha^*,t)$ may be written as a functional integral,

$$P(\alpha,\alpha^*,t)=\int\delta^{(2)}(\alpha-\alpha_t)P'(\alpha_t)d^2\alpha_t, \tag{18}$$

with $P'(\alpha_t)$ a distribution over stochastic paths α_t. Thus, as a generalization of Eq. (11), we propose the *ansatz*

$$\rho(t) = \int P(\alpha,\alpha^*,t)|\alpha\rangle\langle\alpha|\otimes\rho_{T|\alpha}(t)d^2\alpha \tag{19a}$$

$$= \int P'(\alpha_t)|\alpha_t\rangle\langle\alpha_t|\otimes\rho_{T|\alpha_t}(t)d^2\alpha_t, \tag{19b}$$

where $\rho_{T|\alpha_t}(t)$ is the target state conditioned upon α_t.

It follows readily that the *ansatz* satisfies the master equation. The result follows from the one-way coupling (scattering character) as in our comment below Eq. (7): "The result requires the cancelation of $\hat{a}^\dagger\hat{b}$ from $\hat{H}_{ST}$ and the special status of coherent states as eigenstates of $\hat{a}$." Here the cancelation yields a propitious ordering of operators in Eq. (15)—every $\hat{a}$ acting from the left and $\hat{a}^\dagger$ from the right. With this and (19b), the source-target coupling is

$$\mathcal{L}'_{ST}\{|\alpha_t\rangle\langle\alpha_t|\otimes\rho_{T|\alpha_t}(t)\} = -i[\hat{H}_{\rm drive}(\alpha_t),\rho_{T|\alpha_t}(t)], \tag{20}$$

with

$$\hat{H}_{\rm drive}(\alpha_t)=i\sqrt{\gamma_S\gamma_T^f}(\alpha_t^*\hat{b}-\alpha_t\hat{b}^\dagger), \tag{21}$$

where the quantized one-way interaction is replaced by a symmetric interaction of the target qubit with a classical field (recall Mollow [16]). The proof is completed by the further result, using Eqs. (16) and (17),

$$\int\left[\frac{\partial P(\alpha,\alpha^*,t)}{\partial t}|\alpha\rangle\langle\alpha|-P(\alpha,\alpha^*,t)(\mathcal{L}'_S|\alpha\rangle\langle\alpha|)\right]d^2\alpha=0. \tag{22}$$

Substituting ρ in the form (19a) [(19b)] in $d\rho/dt$ and $\mathcal{L}'_S\rho$ [$\mathcal{L}'_T\rho$ and $\mathcal{L}'_{ST}\rho$], we find the master equation solved if $\rho_{T|\alpha_t}$ satisfies the separated target equation

$$\frac{d\rho_{T|\alpha_t}}{dt}=\mathcal{L}'_T\rho_{T|\alpha_t}-i[\hat{H}_{\rm drive}(\alpha_t),\rho_{T|\alpha_t}]. \tag{23}$$

It is important to state in conclusion that Eqs. (19a,b) and (23) separate not only a state, $\rho(t)$, but a stochastic process, fully characterizing the dynamics of the source as a quantum field and the target response. At the level of the master equation, this follows from the Markov assumption, which permits the extension from $\rho(t)$ to correlation functions (Lax's quantum regression [30]). It is particularly clear in quantum trajectories, where, returning to Eqs. (5) and (6), the coherent state factorization is self-consistently preserved if, once adopted, the target qubit quantum jumps are tracked. In this, as eigenstates of $\hat{a}$, coherent states are unique.

Even in an extended system the Schrödinger equation entangles every system part with every other. If, however, one gives up an account at the level of every single quantum the resulting mixed state might factorize. We, thus, separated the state of a laser source and its target. We propose this separability as a criterion to assign the laser state uniquely as a coherent state.

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